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Neural Network Parallel Algorithm for Missile Interceptor Allocation Problem

Kanae Yamaoka, Keisuke Iwai, Hidema Tanaka and Takakazu Kurokawa
Dept. of Computer Science, National Defense Academy of Japan
1-10-20 Hashirimizu, Yokosuka, Japan
{em51051, iwai, hidema, kuro}@nda.ac.jp

***Abstract*—This paper proposes Hopfield type neural network architecture consisting of binary neurons for missile interceptor allocation problem, which is a kind of Weapon Target Assignment Problem, and is a significant problem in military operation field. Through a large number of simulation runs, the proposed neural network architecture could find efficient assignment schedules within several milliseconds. Compared with former works, the developed neural network simulator could find out allocation results with higher quality. Furthermore, its simulation results showed high searching abilities for optimum, or near optimum solutions.**

Keywords—neural network; parallel; binary; missile interceptor; allocation; weapon target;

I. INTRODUCTION

Weapon Target Assignment Problem (WTAP) is to find out the best assigning method of defensive weapon resources to targets under several constraints, and is a significant problem in military operation field [1]. There are many different problems such as the dynamic WTAP [2],[3],[4], and missile interceptor allocation problem [5],[6],[7] classified in WTAP on the standpoint of constraints.

This paper proposes neural network architecture for the missile interceptor allocation problem for city defense. The problem considered in this paper is defined to find out the optimal assignment of N missile interceptors to M cities defending from the incoming target missiles so that the total expected damage per unit incoming missile of all cities should be minimized [8],[9]. The main advantage of the neural network algorithm is that it can be adapted to both special-purpose hardware circuit as well as general-purpose parallel computer to yield feasible solutions rapidly [10], which is suitable to solve the missile interceptor allocation problem. Moreover it has an advantage of its searching ability for optimum, or near optimum solutions. The proposed neural network architecture finds efficient assignment schedules satisfying various physical and operational constraints under given scenarios within several milliseconds.

II. MISSILE INTERCEPTOR ALLOCATION PROBLEM

A. Definition

The problem considered in this paper is defined to find out the best allocation strategy for N missile interceptors to protect M cities from the incoming enemy missile attack so that the total expected damage of all cities per unit incoming missile should be minimized [8]. All cities have different strategic values.

Reference [9] proposed Genetic algorithm (GA) approach to solve this problem. The following four assumptions were applied for the simplicity and for clearness of the procedure.

- The target hitting rate of enemy missile should be 1.

It means if the missile is not intercepted, the city under attack will definitely be destroyed.

- Single shot kill probability of an interceptor should be 1.
- Interceptors allocated to protect a city can intercept missiles whose target should be the exact city.
- There should be no time constraints.

Formulation of the missile interceptor allocation problem is also shown in [9] as follows:

Let N_i be the number of interceptors allocated to the i-th city, and sv_i be the strategic value of the i-th city. If the city is attacked by more than N_i incoming missiles, the city will be destroyed. So that the expected damage value per unit missile for the i-th city can be defined as $sv_i/(N_i+1)$.

The objective of this problem is to minimize the expected damage of unit missile as;

Minimize

$$\sum_{i=1}^{M} \frac{sv_i}{N_i+1}, \tag{1}$$

subject to the constraints

$$\sum_{i=1}^{M} N_i = N, \tag{2}$$

$$N_i \geq 0 \text{ for } i = 1,2,\ldots,M.$$

Reference [9] proposed a Genetic algorithm (GA) combined with Tabu search algorithm based local search method. In this paper, we call this algorithm as “GA+TS”. In general, there have been so many solution methods to solve WTAP, such as Genetic algorithms [2],[3],[9],[11], simulated annealing [12], learning [6],[7], Tabu search [4],[9], Linear integer programming [13], Neural network architecture [1], and

Neuro-Dynamic Programming [5]. This paper proposes neural network architecture consisting of binary neurons with Hopfield type network because of its advantages to be adapted to both special-purpose hardware circuit as well as general-purpose parallel computer to yield feasible solutions rapidly [10], and for its searching ability for optimum, or near optimum solutions.

B. Example problems

Reference [9] discussed the performance of GA+TS algorithm through the evaluation of computational simulation using three different sizes of missile interceptor allocation problems. These example problems as well as the best solutions presented in [9] are summarized in Table 1. Example problem 1 is a problem to find out the best allocation of 100 missile interceptors to 15 cities. The strategic values of cities are listed on the fourth column of Table 1. The best solution found by the GA+TS algorithm is listed on the next column. Then total expected damage of unit missile can be calculated as 8/(5+1)+5/(3+1)+…+14/(6+1) = 27.5444. The second and third example problems assume 25 cities – 300 interceptors, and 40 cities – 500 interceptors respectively. Their strategic values and the best solutions are also listed on this table.

III. NEURAL NETWORK ARCHITECTURE

In this section, we propose neural network architecture to solve the missile interceptor allocation problem. As defined in the previous section, this problem is to find out the best allocation of N interceptors to M cities so as to minimize the total expected damage of unit missile. We prepare a two dimensional Hopfield type neural network array as shown in Figure 1. In this figure, each row accords with a city numbered 1 to M, while each column accords with a missile interceptor numbered 1 to N. Each square accords with a binary neuron whose input/output function at time t is given by

$$V_{ij}(t)=1,\ \text{if}\ U_{ij}(t)>0 \tag{3}$$

$$0,\ \text{otherwise for}\ i=1,2,\ldots,M,\ j=1,2,3,\ldots,N.$$

Note that $U_{ij}(t)$ and $V_{ij}(t)$ are the input and the output of the ij-th neuron (i=1,2, …, M, j=1,2,…,N) in the neural network array respectively. Hopfield type neural networks consisting of binary neurons have splendid abilities to solve combinatorial optimization problems [10],[14],[15]. Moreover it has an advantage of its searching ability for optimum, or near optimum solution. In Figure 1, black squares accord with the firing neuron. For example, the black square in the 1-st row, 5-th column means that the according binary neuron is firing to indicate that the 5-th missile interceptor is allocated to the 1-st city.

The output of the ij-th neuron $V_{ij}(t)$ (i=1,2, …, M, j=1,2,…,N) in the neural network array is given by the following motion equation:

$$\frac{dU_{ij}(t)}{dt}=-Af\left(\sum_{\substack{k=1\\k\neq i}}^{k=M}V_{kj}(t)\right)V_{ij}(t)+Bg\left(sv_i,\sum_{k=1}^{k=N}V_{ik}(t)\right)\left(1-V_{ij}(t)\right). \tag{4}$$

We prepare negative synaptic connections between the two neurons in the same column according to the first term in the equation (4), and positive synaptic connections in the same row according to the second term. Coefficients A and B are constant positive integers. The function $f(x)$ becomes 1 if x>0, and 0 otherwise. The function $g(x, y)$ returns the integer value of x/y. The first term in equation (4) is an inhibitory force in order to avoid more than two neurons in the same column "j" to be fired at the same time. This means that one missile should be allocated to only one city. The second term is an excitatory force to fire the ij-th neuron according to the strategic value of

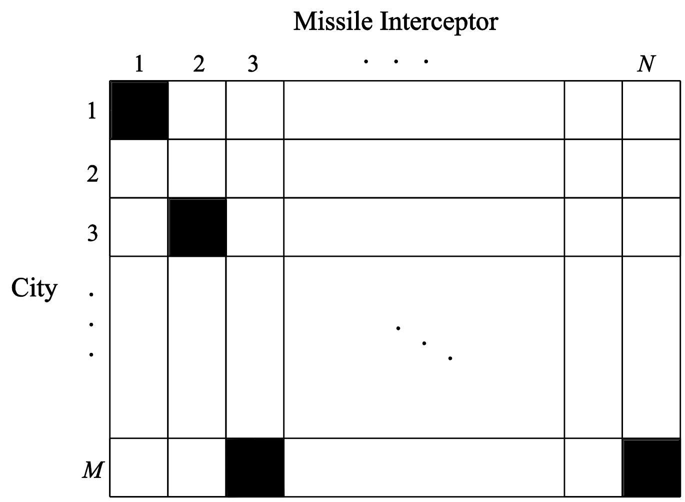

Fig. 1. Neural network representation for a missile interceptor allocation problem.

TABLE I. EXAMPLE PROBLEMS

Example problems	# of city	Total # of missile interceptor	Total expected damage of unit missile	Strategic values of cities	# of missile interceptors allocated to cities
1	15	100	27.5444	8,5,15,7,16,15,8,9,6,32, 30,25,21,16,14	5,3,7,4,8,7,5,5,4,11, 10,10,8,7,6
2	25	300	30.0215	7,9,21,3,8,11,12,9,20,18, 17,15,11,14,10,17,32,31,29,13, 12,25,24,19,26	8,9,15,6,8,10,11,9,15,13, 12,11,9,11,9,12,18,17,17,11, 10,15,15,14,15
3	40	500	49.701	7,9,21,3,8,11,12,9,20,18, 17,15,11,14,10,17,32,31,29,13, 23,22,21,18,14,11,29,9,4,17, 28,25,15,33,6,20,10,24,32,12	8,9,15,6,8,10,11,9,15,13, 12,11,9,11,9,12,18,17,17,11, 14,14,14,14,11,10,17,9,5,12, 16,16,12,18,8,14,9,15,18,10

the *i*-th city sv_i, and also to the number of the fired neurons in the *i*-th row when the *ij*-th neuron is not firing.

The following procedure describes the parallel computation of the proposed neural network.

[Step 1] Set $t = 0$.

[Step 2] The states of inputs $U_{ij}(t)$ (i=1,2,…,M, j=1,2,…,N) are set to random negative number.

[Step 3] Evaluate values of $V_{ij}(t)$ based on the binary function from $U_{ij}(t)$.

[Step 4] Use the motion equation (4) to compute $\Delta U_{ij}(t)$.

[Step 5] Compute $U_{ij}(t+1) = U_{ij}(t) + \Delta U_{ij}(t)$.

[Step 6] Increment t by 1.

[Step 7] If exactly one neuron fires in each column then terminate this procedure, else go back to Step 3.

IV. Evaluation

We developed a software simulator for the proposed neural network using C language on RHEL5 Server (Xeon X5680, 3.33GHz, 94GB). Its computational performance is evaluated through the following three example problems (15 cities – 100 interceptors, 25 cities – 300 interceptors, and 40 cities – 500 interceptors) shown in section II.*B*. In each example problem, one million simulation runs were performed. In each simulation run, initial state of the system was randomly generated.

Table 2 shows the comparison between the results of the developed neural network simulator with GA+TS [Han04] on the standpoint of the quality of the allocation results. Apparently our neural network simulator could find out a higher quality for the allocation results compared with GA+TS. The allocation results were affected by the initial state of the neurons.

Table 3 summarizes the computational performance of the developed neural network simulator. Average number of iteration steps requiring for the neural network simulator to converge to a solution for example problems were 17.25, 19.90 and 17.91 respectively. While success ratio for the three example problems were all 100%. In average, it required 0.36ms, 1.94ms, and 5.57ms for each simulation run respectively.

Table 4 compares the details of the solutions found by GA+TS and NN. In case of example problem 3, NN could find out two different optimal solutions, which are denoted by NN1 and NN2. This fact shows the superiority of the binary neural network to find out optimum solution.

Table 5 is a list of Top 5 solutions found by the developed

TABLE II. Comparison of the Algorithms on the Standpoint of the Quality of the Allocation Results

Example problems	Algorithms	Total expected damage of unit missile		
		Best	Worst	Average
1	GA+TS	27.5444	27.6786	27.5966
	NN	27.4939	27.5308	27.4967
2	GA+TS	30.0215	30.2219	30.1067
	NN	29.9665	29.9690	29.9666
3	GA+TS	49.701	50.391	50.029
	NN	47.2652	47.2961	47.2656

TABLE III. Computational Performance of the Developed Neural Network Simulator

Example problems	Average # of iteration	Average computational time	Success ratio
1	17.25	0.36msec	100%
2	19.90	1.95msec	100%
3	17.91	5.57msec	100%

TABLE IV. Comparison of the Solutions

Example problems	Algorithms	# of missile interceptors allocated to cities	Sum of expected damage of unit missile
1	GA+TS	5,3,7,4,8, 7,5,5,4,11, 10,10,8,7,6	27.5444
	NN	5,4,7,4,7, 7,5,5,4,11, 10,9,8,7,7	27.4939
2	GA+TS	8,9,15,6,8, 10,11,9,15,13, 12,11,9,11,9, 12,18,17,17,11, 10,15,15,14,15	30.0215
	NN	8,9,14,5,8, 10,10,9,14,13, 13,12,10,11,9, 13,18,17,17,11, 10,15,15,13,16	29.9665
3	GA+TS	8,9,15,6,8, 10,11,9,15,13, 12,11,9,11,9, 12,18,17,17,11, 14,14,14,14,11, 10,17,9,5,12, 16,16,12,18,8, 14,9,15,18,10	49.701
	NN1	8,9,15,5,9, 10,11,9,14,13, 13,12,10,12,10, 13,18,18,17,11, 15,15,14,13,12, 10,17,9,6,13, 17,16,12,18,7, 14,10,16,18,11	47.2652
	NN2	8,9,14,5,9, 10,11,9,14,13, 13,12,10,12,10, 13,18,18,17,11, 15,15,15,13,12, 10,17,9,6,13, 17,16,12,18,7, 14,10,16,18,11	47.2652

TABLE V. TOP 5 SOLUTIONS FOUND BY THE DEVELOPED NEURAL NETWORK SIMULATOR

TOP5	**Example Problems**		
	1	**2**	**3**
1	27.4939 701,603	29.9665 988,448	47.2652 871,813
2	27.5030 298,299	29.9690 11,552	47.2677 110,533
3	27.5308 98	-	47.2741 15,139
4	-	-	47.2821 1,921
5	-	-	47.2797 293

neural network simulator. The Upper rows present the sum of expected damage of unit missile for each solution. And the lower rows present the frequency of the solution. In case of example problem 1, the simulator could find out the optimum solution with the highest frequency as 701,603 out of one million simulation runs. Other example problems 2 and 3 also showed the highest frequency at the optimum solutions. In cases of example problem 1 and 2, all solutions found by the simulator could be classified to three and tow values only respectively.

V. CONCLUTION

This paper proposed a neural network architecture for missile interceptor allocation problem. This problem is a kind of Weapon Target Assignment Problem, which is a significant problem in military operation field. Through a large number of simulation runs, the proposed neural network architecture could find efficient assignment schedules within several milliseconds. Their qualities are much higher than those by GA+TS. Furthermore, its simulation results showed high searching abilities for optimum, or near optimum solutions. Development of neural network hardware system using ASICs of FPGA chips to yield feasible solutions rapidly will be the future work.

With a slight modification, the proposed neural network architecture can accord with more realistic assumptions that the single shot kill probability should become a decimal fraction. Furthermore, the proposed neural network architecture can be assigned to other WTAPs in military operation field.

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